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The asymptotic behaviour of singular solutions to the solutions of linear partial differnential equatiions in the complex domain

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§1. Let $L(z,\partial_z)$ be a linear partial differential operator with the order $m\geq 1$, whose coefficients are holomorphic functions in a neighbourhood $\Omega=\{z\in\mathbb{C}^{n+1};\ |z|\leq R\}$ of $z=0$ in $\mathbb{C}^{n+1}$, where $z=(z_0,z_1,\ldots,z_n)=(z_0,z')$ and $|z|=\max_{0\leq i\leq n}|z_i|$. Let K be a nonsingular hypersurface through $z=0$. For the simplicity we choose the coordinate so that $K=\{z_0=0\}$. In the following we consider the equation

(1.1) $\quad L(z,\partial_z)u(z)=f(z),$

where $u(z)$ may have singularities on K, and $f(z)$ is holomorphic in Ω.

We introduce some function spaces and the definitions. $\Omega(a,b)$ is the set defined by $\Omega(a,b)=\{z;\ a<\arg z_0<b;\ |z|\leq R\}$ and $\Omega'=\Omega\cap\{z_0=0\}$. Firstly we define the function spaces:

$\mathcal{O}(\Omega)=\{f(z);\ f(z)$ is holomorphic in $\Omega\}$, $\mathcal{O}(\Omega')=\{f(z');\ f(z')$ is holomorphic in $\Omega'\}$ and $\tilde{\mathcal{O}}(\Omega(a,b))=\{f(z);\ f(z)$ is holomorphic in $\Omega(a,b)\}$. If $b-a>2\pi$, $\tilde{\mathcal{O}}(\Omega(a,b))$ contains multi-valued functions. We define other function spaces.

Definition 1.1. $\tilde{\mathcal{O}}_{(\gamma)}(\Omega(a,b))=\{f(z)\in\tilde{\mathcal{O}}(\Omega(a,b));$ for any a',b' with $a<a'<b'<b$ and $\varepsilon>0$, there is a $C_{\varepsilon,a',b'}$ such that

(1.2) $\quad |f(z)|\leq C_{\varepsilon,a',b'}\exp(\varepsilon|z_0|^{-\gamma})\quad$ in $\Omega(a',b')$.

Definition 1.2. $f(z)\in\tilde{\mathcal{O}}(\Omega(a,b))$ is said to have the γ-asymptotic expansion in $\Omega(a,b)$, if for any N

(1.3) $\quad |f(z)-\sum_{k=0}^{N-1}a_k(z')(z_0)^k|\leq AB^N\Gamma(N/\gamma+1)|z_0|^N$

holds in $\Omega(a',b')$ for any a',b' with $a<a'<b'<b$, where $a_k(z')\in\mathcal{O}(\Omega')$

and A and B are some constants. The totality of functions with the γ-asymptotic expansions is denoted by $Asy_{\{\gamma\}}(\Omega(a,b))$.

Secondly we define characteristic indices ([1],[2]): We write $L(z,\partial_z)$ in the following form.

$$(1.3)\qquad \begin{aligned} L(z,\partial_z) &= \sum_{k=0}^{m} L_k(z,\partial_z), \\ L_k(z,\partial_z) &= \sum_{\ell=s_k}^{k} A_{k,\ell}(z,\partial')(\partial_0)^{k-\ell}. \end{aligned}$$

$L_k(z,\partial_z)$ is the homogenous part of the degree k. We expand $A_{k,\ell}(z,\partial')$ at $z_0=0$, $A_{k,\ell}(z,\partial')=(z_0)^j a_{k,\ell}(z,\partial')$, $j=j(k,\ell)$, where if $A_{k,\ell}(z,\partial')\not\equiv 0$, $a_{k,\ell}(0,z',\partial')\not\equiv 0$. Thus we have

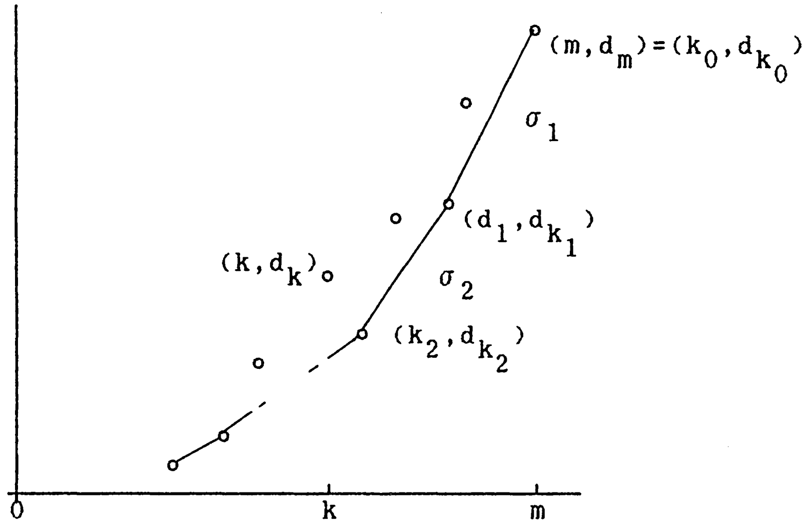
$$(1.4)\qquad L_k(z,\partial_z)=\sum_{\ell=s_k}^{k}(z_0)^j a_{k,\ell}(z,\partial')(\partial_0)^{k-\ell},$$

Define

$$(1.5)\qquad d_k=\min\{\ell+j(k,\ell);\ A_{k,\ell}(z,\partial')|_{z_0=0}\not\equiv 0\}.$$

Put $A=\{(k,d_k)\in\mathbb{R}^2; 0\le k\le m,\ d_k\ne+\infty\}$. Let $\hat{A}$ be the convex hull of A, Σ be the lower convex part of the boundary of $\hat{A}$ and Δ be the set of vertices of Σ. Δ consists of finite points: $\Delta=\{(k_i,d_{k_i}); i=1,2,..p'\}$, $m=k_0>k_1>\dots>k_{p'}\ge 0$. We put

$$(1.6)\qquad \sigma_i=\max\{1,\ (d_{k_{i-1}}-d_{k_i})/(k_{i-1}-k_i)\}.$$



Then there is a $p\in\mathbb{N}$ such that $\sigma_1>\sigma_2>\dots>\sigma_p=1$. We call $\{\sigma_i\}$ $(1\le i\le p)$ characteristic indices.

§ 2. By using the definition in § 1 we can state Theorem:

Theorem. Assume

(a) $\sigma_1>1$, (b) $d_{k_{p-1}}=0$ and (c) $d_{k_i}=s_{k_i}$ $(0\le i\le p-2)$.

Let $u(z)\in\tilde{\mathcal{O}}(\Omega(a,b))$ $(b-a>\pi)$ be a solution of $L(z,\partial_z)u(z)=f(z)\in\mathcal{O}(\Omega)$. If $u(z)\in\tilde{\mathcal{O}}_{(\gamma)}(\Omega(a,b))$ $(\gamma=\sigma_{p-1}-1)$, then $u(z)\in Asy_{\{\gamma\}}(\Omega(a,b))$, that is, $u(z)$ has the γ-asymptotic expansions in $\Omega(a,b)$.

This is a generalzation of the theorem in [3].

References

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[2] S.ŌUCHI: Index, localization and classification of characteristic surfaces for linear partial differential operators, Proc. Japan Acad. Ser. A Math. Sci. 60 (1984), 189-192.

[3] S.ŌUCHI: Vanishing of singularities of solutions and an integral representation of singular solutions of linear partial differential equations, Calcul d'operateurs et fronts d'ondes (ed. by J.Vaillant), Hermann, Paris (1988) 168-177.